

MTH 111, Math for Architects, Final Review, Spring 2014

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QUESTION 1. a) Find an equation of the plane that contains the point $Q = (1, 2, 0)$ and the line L that has parametric equations $L : x = 1 + 3t, y = 5 + t, z = 1 + 4t$

b) Find the distance between Q and L.

c) Choose any two points on the line L, say Q_1, Q_2. Find the area of the triangle Q_1Q_2Q.

QUESTION 2. a) Given $V = i + 2j + 2k$. Find a vector F that is parallel to V such that $|F| = 7.25$

b) Given $V = 3i - 4j$ and $W = 2i + 2j + k$. Find $Proj_W^V$ and $|Proj_W^V|$. If θ is the angle between V and W what is $cos(\theta)$?

c) A particle moves on the ellipse $2x^2 + 5y^2 + 4x + 10y = 70$. The y is decreasing at rate 0.5 cm/sec. Find the rate of change of x at the point (3, 2).

d) Find the vertex, the directrix and the focus for $9y = x^2 - 10x - 11$ and then sketch .

QUESTION 3. a)Given that an ellipse is centered at (2, 4), it has constant $k = 10$ and one of the foci is $(5, 4)$. Write down the equation of the ellipse and then sketch the ellipse.

b) Find the equation of the hyperbola that has $(6, 4), (-2, 4)$ as its foci, and one of its vertices is $(4, 4)$.

QUESTION 4. a) Given the points: $A = (2, 3)$ and $B = (6, 6)$. Find a point C on the line $y = 2$ so that $|AC| + |CB|$ is minimum. You need to find the coordinates of the point C.

b) Find the absolute maximum value of y and the absolute minimum value of y for $y = (x^2 - 3x + 1)e^x$ defined on $[-2, 2]$ (i.e., $-2 \le x \le 2$)

c) Find two numbers x, y where $x + 4y = 20$ and xy is maximum. SHOW THE WORK

QUESTION 5. a) Find $lim_{x\to 5} \ \frac{\sqrt{3x-6}-3}{x^2+x-30}$

b) $Lim_{x\to -2} \ \frac{ln(3x+7)}{e^{(x+2)}-2x-5}$

c) Let $f(x) = e^{2x-3} + 2\sqrt{8x-8} + ln(6x-8) + 4$. Find the equation of the tangent line to the curve of $f(x)$ when $x = 1.5$.

QUESTION 6. a)Given $xe^{y-3} + ln(y + x - 4) + yx + y + x - 13 = 0$; also given $(2, 3)$ lies on the curve. You have been asked to approximate the y value when $x = 1.6$, what will you do? SHOW ALL THE WORK AND APPROXIMATE the y value when $x = 1.6$.

b) We want to construct a rectangle with maximum area inside the ellipse $y^2 + 4x^2 = 20$ such that two vertices on the x-axis and the other two vertices on the upper half of the ellipse. What should be the length and the width of such rectangle? SHOW ALL THE WORK.

QUESTION 7. Evaluate the following integrals:

a) $\int \ 7e^{x+1} + \sqrt{x+1} + 4x^e \ dx$

a/2) $\int \ \frac{6x^2+18}{x^3+9x+3} \ dx$

a/3) Find the area of the region that is bounded by $f(x) = -x^2 + 3x + 5$ and the line $y = x + 2$ where $0 \le x \le 4$.

QUESTION 8. (i) Let $f(x) = -x^2 + 8x - 1$. The slope of the tangent line to the curve at the point $(1, 5)$

a. -2
b. 6
c. 5

(ii) Let $f(x) = -x^3 + 12x + 1$. Then $f(x)$ increases on the interval

a. $x \in (-2, 2)$
b. $x \in (-\infty, -2) \cup (2, \infty)$
c. $x \in (-\sqrt{12}, \sqrt{12})$
d. none of the above

(iii) let $f(x) = 3e^{(x^2-2x)} + 4$. Then $f'(2)$

a. 3
b. 6
c. 2
d. none of the above

(iv) Let $f(x) = xe^{(x-2)} + e^{(x-2)} + 3$. Then

a. $f(x)$ has a local maximum at $x = 2$
b. $f(x)$ has a local minimum at $x = -2$
c. $f(x)$ has a local minimum at $x = -1$
d. $f(x)$ has a local maximum at $x = -1$
e. none of the above

(v) Let $f(x) = -x(x-18)^5$. Then

a. $f(x)$ has a local minimum at $x = 18$
b. $f(x)$ has a local maximum at $x = 3$
c. $f(x)$ has a local maximum at $x = 18$
d. $f(x)$ has a critical value when $x = -18$
e. none of the above

(vi) Given $x^2 + y^2 - xy = 0$. Then $dy/dx =$

a. $\frac{y-2x}{2y-x}$
b. $\frac{2y-x}{y-2x}$
c. $\frac{y-2x}{x-2y}$
d. $\frac{2x-y}{2y-x}$

(vii) Given $f(x) = \sqrt{4x-3} + \frac{1}{x} + 2$. Then $f'(1) =$

a. 4
b. 2
c. 3
d. 1

(viii) Given the curve of $f'(x)$. Then

a. $f(x)$ is decreasing on the the interval $(1, 2)$

b. $f(x)$ is decreasing on the interval $(-\infty, 0)$

c. $f(x)$ is decreasing on the interval $(1, \infty)$

d. $f(x)$ is increasing on the interval $(-\infty, 2)$

e. above, there are more than one correct answer.

(ix) Using the curve of $f'(x)$ above. Then

a. $f(x)$ has a local min. value at $x = 0$ but no local max. values.

b. $f(x)$ has neither local min. values nor local max. values

c. $f(x)$ has a local min. value at $x = 0$ and a local max. value at $x = 1$.

d. $f(x)$ has a local max. value at $x = 2$

(x) Using the curve of $f'(x)$ above. Then

a. the curve of $f(x)$ must be concave down on the interval $(0, 1)$.

b. the curve of $f(x)$ must be concave down on the interval $(-\infty, -1)$

c. the curve of $f(x)$ must be concave up on the interval $(2, \infty)$

d. above, there are more than one correct answer.

(xi) Given $f'(3) = f'(-1) = f'(6) = 0$, $f^{(2)}(2) = 4$, $f^{(2)}(-1) = -5$, and $f^{(2)}(6) = 0$ (note that $f^{(2)}$ means the second derivative of $f(x)$). Then

a. $f(x)$ has a local max. value at $x = -1$.

b. $f(x)$ has a local max. value at $x = 3$

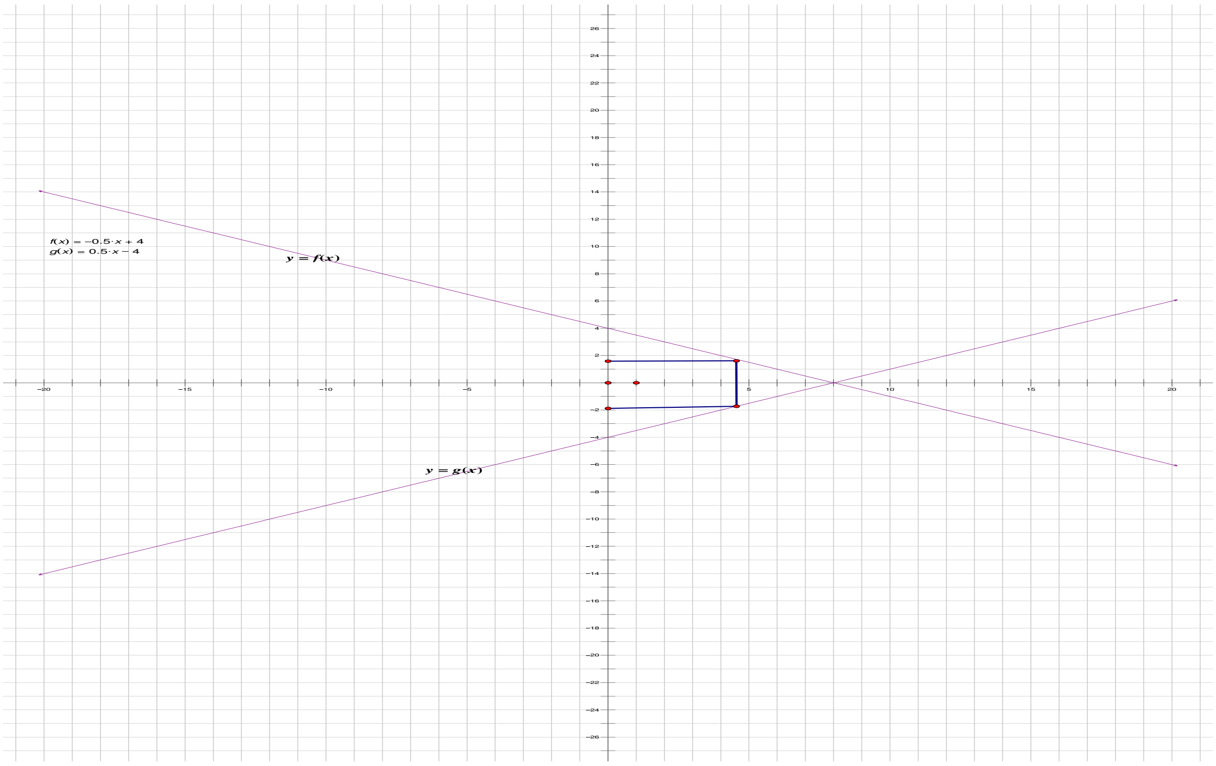
c. $f(x)$ has neither local min. value nor local max. value at $x = 6$.

d. None of the above

(xii) Given x, y are two positive real numbers such that $x + 2y = 26$ and xy is maximum. Then $xy =$

a. 52
b. 78
c. 84.5
d. 169
e. none of the above

(xiii) What is the area of the largest rectangle that can be drawn as in the figure below (note $f(x) = -0.5x + 4$ and $g(x) = 0.5x - 4$)?



a. 64
b. 32
c. 16
d. none of the above

(xiv) Given the points $A = (2, 4)$ and $B = (0, 6)$. What is the point c on the $x-axis$ so that $|AC|+|CB|$ is minimum?

a. (2, 0)
b. (1.5, 0)
c. (1.2, 0)
d. (1, 0)
e. None of the above

(xv) A particle moves on the curve $4x^2 + 6y^2 = 22$. If the x-coordinates increases at rate 0.3/second, what is the rate of change of y when the particle reaches $(2, 1)$?

a. 0.4
b. -0.3
c. -0.4
d. none of the above

(xvi) Given $f(x) = (4x - 7)^{11}$, $f'(2)$ =

a. 11
b. 4
c. 44
d. non of the above

(xvii) Given $f(x) = ln[\frac{5x-14}{3x-8}]$. Then $f'(3)$

a. $\frac{5}{3}$
b. 15
c. 2
d. None of the above

(xviii) Given $(-4, 2), (0, 0), (6, 8)$ are vertices of a triangle. The area of the triangle is

a. 22
b. 44
c. $\sqrt{44}$
d. $\sqrt{22}$
e. None of the above.

(xix) $lim_{x \to 2} \ \frac{e^{(3x-6)}+x-3}{x^3-x^2-4} =$

a. 0
b. 0.5
c. 0.25
d. none of the above

(xx) $lim_{x \to 3} \ \frac{x^2-18}{(x-3)^2} =$

a. $-\infty$
b. 0
c. ∞
d. DNE (does not exist)
e. -9

QUESTION 9. Find an equation of the ellipse with the vertices $(4, 3), (1, 7)$, and $(-2, 3)$. Find the constant k. Find the foci. Make a rough sketch of such ellipse.

QUESTION 10. Find an equation of the hyperbola that is centered at $(2, 1)$ and with constant $k = 6$ such that (2, 6) is one of the foci. Find the second foci, find the vertices, and make a rough sketch of such hyperbola.

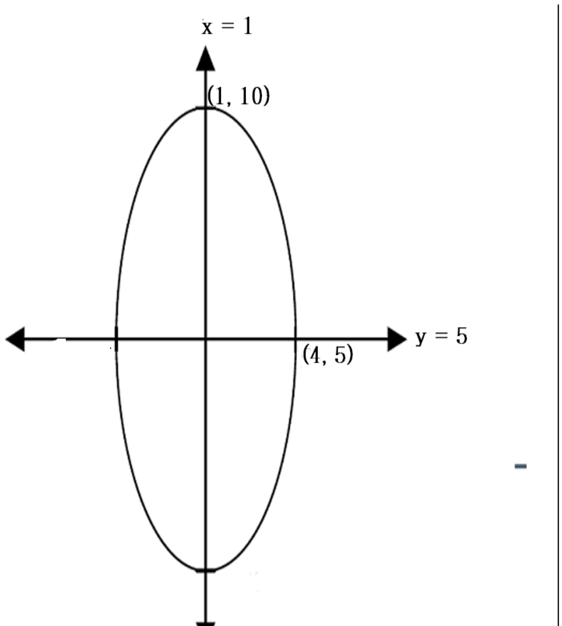
QUESTION 11. Given $x = 1$ is the directrix line of a parabola that passes through the point $(6, 5)$ and the line $y = 2$ passes through the vertex of the parabola. Find the vertex, the focus, and make a rough sketch of such parabola. Then find an equation of the parabola. [Hint: there are two such parabolas, just find one]

QUESTION 12. Find the directrix, the focus, and the vertex of the parabola $y = 0.5(x + 5)^2 + 4$

QUESTION 13. Find the foci, the constant k, and the vertices of the ellipse $(x + 2)^2/25 + (y - 3)^2/9 = 1$

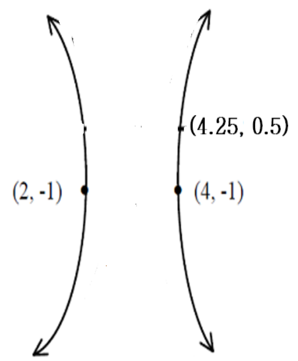
QUESTION 14. Find the center, the foci, the vertices of the hyperbola $x^2 - 2y^2 - 4y = 18$

QUESTION 15. Find the foci, and the equation of the below ellipse:



QUESTION 16.

Find the foci, and the equation of the below hyperbola:



QUESTION 17. Find an equation of the plane P that contains the line $L : x = t, y = 1 - t, z = 2t$ and the point $Q = (1, 0, 5)$ [note that the point Q does not lie on L]

QUESTION 18. a) Find the distance between the point $Q = (2, 2, 1)$ and the plane $x + 3y + 5z = 15$

b) The line $L_1 : x = 5t, y = 4 - t, z = 3 + t$ intersects the line $L_2 : x = 1 + 2s, y = 9 - 3s, z = 2s$ at a point Q. Find Q

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